### UNIVERSIDAD ADOLFO IBÁÑEZ

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#### UNIVERSIDADE PRESBITERIANA MACKENZIE

PROGRAMA DE PÓS-GRADUAÇAO EM ENGENHARIA ELÉTRICA E COMPUTAÇÃO

## DYNAMICS OF AUTOMATA NETWORKS: THEORY AND NUMERICAL EXPERIMENTS

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